Supplemental Material

We created a validated circular shape space comparable to the commonly used "color wheel" (Ma, Husain, & Bays, 2014; McDermott & Webster, 2012). One conceptualization of this color wheel can be as a set of "prototype" colors (e.g. red, blue, green) separated by the blended versions of those prototypes (e.g. purple is the result of blending red and blue). Accordingly, we designed a set of 2D line drawings to act as prototype shapes, morphed between pairs of these prototypes to generate a space of 360 stimuli, then reconstructed subjective shape space using MDS. The reconstruction of shape space allowed us to identify problematic portions of the space that were not circular, with problematic regions either entirely removed or completely redesigned. This procedure was then iteratively repeated until a circular shape space could be obtained. *Experiment 1* in the main text describes the final validation iteration that ensured VCS space was circular. Here, the *Supplemental Material* describes step-by-step all of the earlier validation iterations, as well as provides information regarding the simulated behavior of the circularity measure (see *Step 5. Assessing circularity*). Note that although the circularity score was calculated for the aligned shape space in the main text, we also calculated the circularity score for individual group shape spaces (*Figure 3d*). Some validation iterations have only a single group, as we did not always conduct the alignment procedure.

Circularity Simulations

The quantitative measure of circularity describing the geometry of reconstructed space was based on isoperimetric inequality, a geometric inequality that captures the relationship between area and perimeter of a spatial configuration (Osserman, 1978). The circle is the shape with largest area relative to perimeter in two dimensions, such that larger values of C indicate a geometry that is closer to a perfect circle. To our best knowledge, we are the first group to quantitatively describe the geometry of a reconstructed space. Indeed, it is currently standard practice for experimenters to qualitatively assess MDS results (Hout, Goldinger, & Ferguson, 2013; Hout, Papesh, Goldinger, 2012; Hout et al., 2016; Kriegeskorte, Mur, & Bandettini, 2008; Shepard, 1980).

A threshold of C = 0.90 was selected to determine when a reconstructed geometry was sufficiently circular (see *Step 5. Assessing circularity*). To confirm the robustness of this threshold, we simulated the behavior of C in two ways. First, 15 coordinates were randomly placed into two dimensional space one million times. No C values greater than 0.40 were observed, suggesting that C values higher than 0.40 cannot be obtained from random chance alone. Second, we started with an idealized perfectly circular space in order to observe a larger distribution of C values. Random noise was added to each coordinate in two dimensional space one million times. A value of C = 0.90 indicated a geometry that is smooth and nearly a perfect circle. The stimuli were always in the correct order and the reconstructed space typically had minor bumps or ridges, with some minor deviation in position from an expected circle. Fully commented code to run these simulations are available on the Open Science Framework (https://osf.io/d9gyf). See *Figure S1* for pictorial examples of simulated geometries at a range of C values.

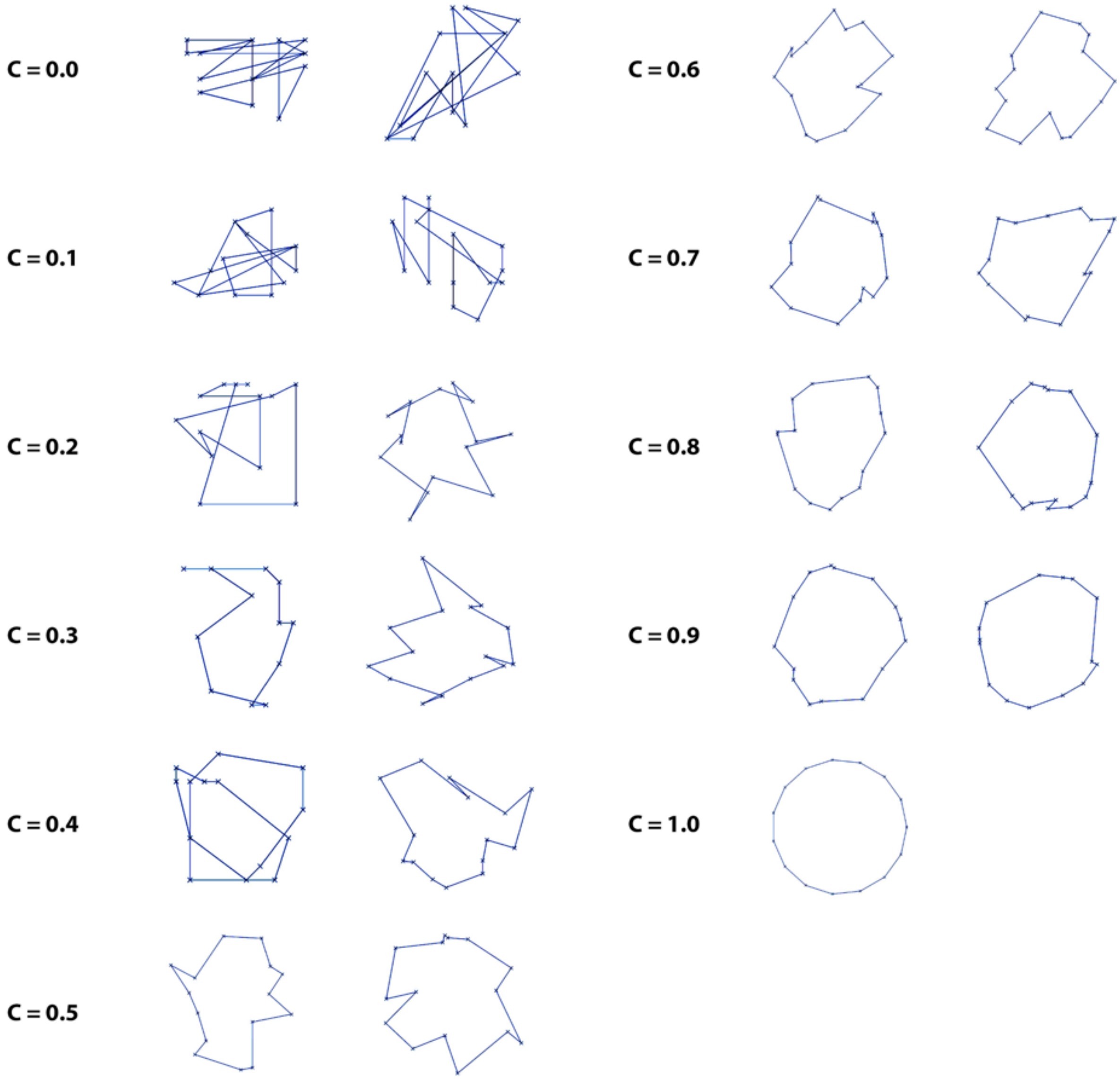
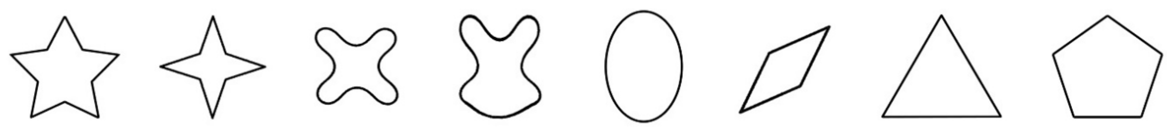


Figure S1. Simulated geometries at different circularity values. *C* near 0.0 indicates a geometry with many overlapping edges. As the *C* value approaches 1.0, geometries become increasingly circular.

Validation Iteration 1

A set of 8 prototype shapes was manually designed in Photoshop (*Figure S2*). These shapes were placed on equidistant points along a 2D circle, specifically designed so that all shapes were visually distinct and so that the change in visual similarity was approximately uniform. Nine participants (M = 21.33 years, SD = 2.24 years, *Females* = 6) were recruited to rate the similarity of this first set of prototype shapes, requiring 184 experimental trials. All combinations of shape pairs were rated relative to each shape in the set, such that each unique pair of shapes was rated 12 times.

a. Prototype Shapes



b. Morphed Space

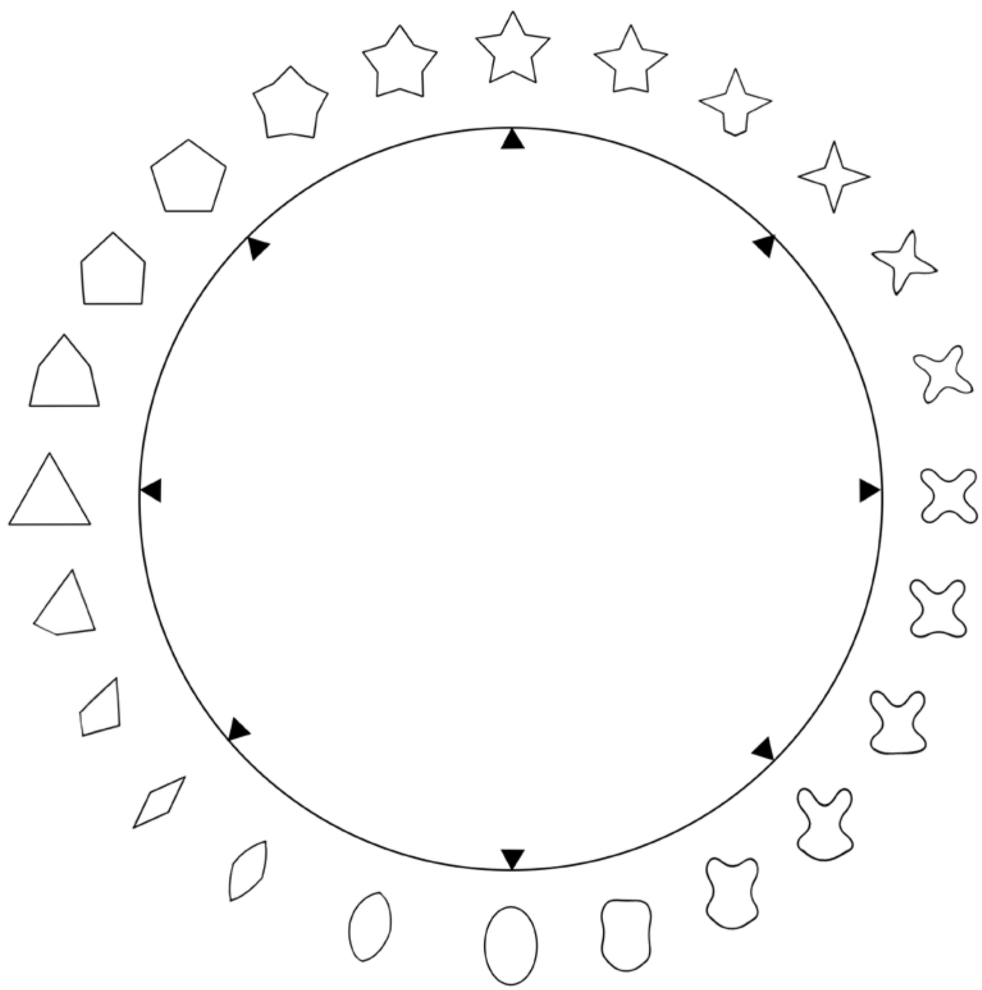


Figure S2. The first validation iteration. (*a*) The set of eight prototype shapes. (*b*) An example of a shape space generated by morphing between pairs of prototype shapes. Triangles represent the position of the rated shapes on circular space, which were the same as the prototype shapes used to build this space.

Results. C was 0.692 ($p < 0.001$), suggesting that the first iteration of VCS space was not circular. Qualitatively, the similarity matrix (*Figure S3a*) and MDS reconstructed shape space (*Figure S3b*) revealed that the eight rated shapes were confounded by visual similarity.

Turning to the similarity matrix, the shape positioned at 181 degrees was dissimilar relative to most shapes in the set, even those that were positioned close in angular distance. For example, the similarity rating between the shapes located at 181 and 226 degrees was 0.63, even though they were very close in angular distance. Turning to the MDS plot, certain shapes with pointed angles were clustered very closely with each other in a way that did not fit with an expected circular space. Note that although this iteration of VCS space was not circular, the homogeneity score was found to be high (0.76; $p < 0.0001$), suggesting that participants had similar shape spaces compared to each other.

a. **b.**

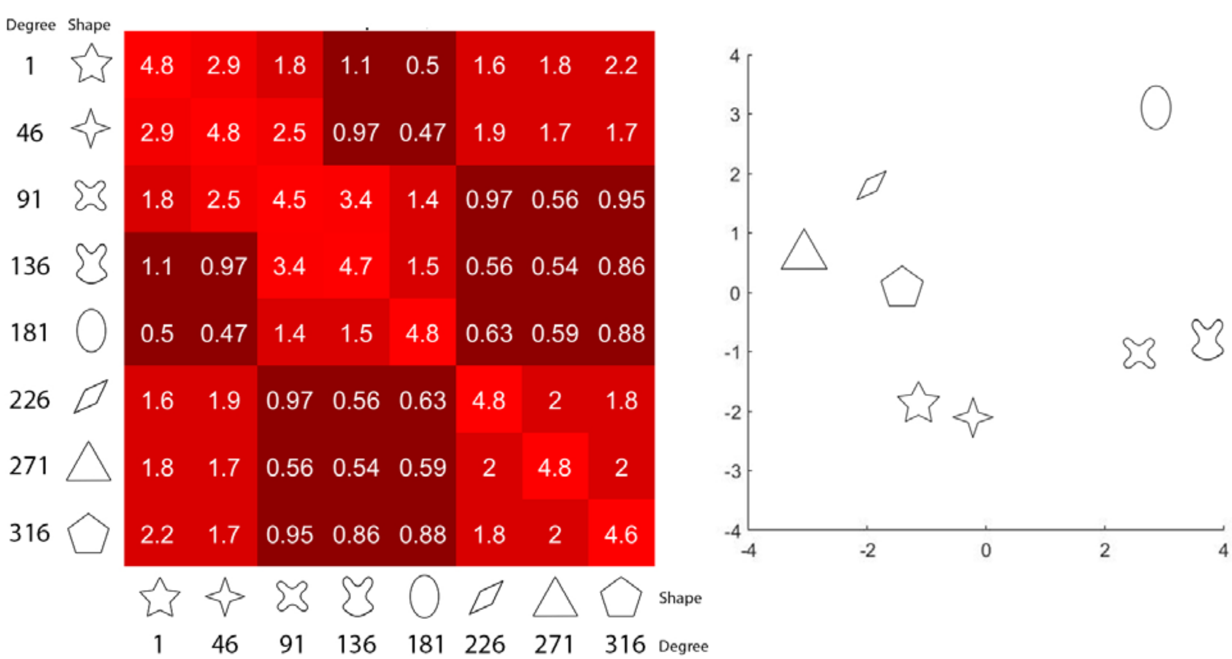


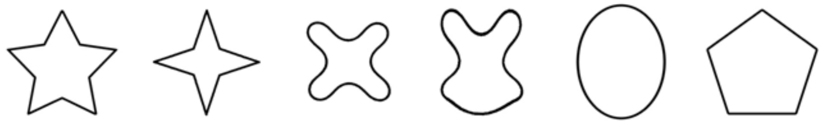
Figure S3. Results from the first validation iteration. (*a*) Similarity matrix of averaged similarity ratings from 0 being "*no similarity*" to 5 being "*identical*". (*b*) MDS revealed that the subjective shape space was not circular, with the shapes with pointed angles especially problematic.

Validation Iteration 2

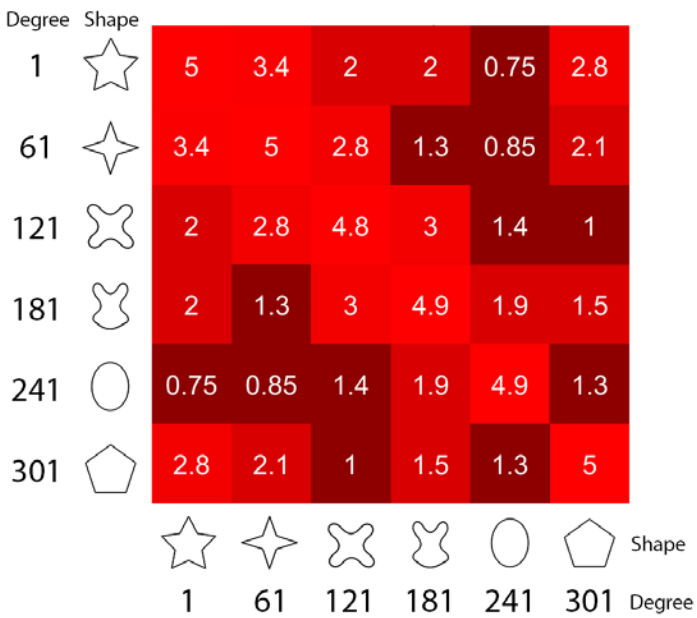
The only difference between this validation iteration and the previous one was the removal of two problematic shapes (*Figure S4a*), with all other shapes being identical. Six participants (M_{age} = 20.83 years, SD_{age} = 1.72 years, *Females* = 4) were recruited. With six rated shapes in the set, the experiment required 72 trials. All combinations of shape pairs were rated relative to each shape in the set, such that each unique pair of shapes were rated 8 times.

Results. C was 0.749 ($p < 0.01$), suggesting that this validation iteration was more circular than the first validation iteration. However, this iteration was not considered circular as it did not meet the selected threshold of $C = 0.90$. Qualitatively, both the similarity matrix (*Figure S4b*) and resultant MDS solution (*Figure S4c*) revealed that the overall space was confounded by similarity. Turning to the similarity matrix, the shape positioned at 241 degrees was dissimilar to most other shapes in the set. Turning to the MDS plot, the shape positioned at 241 degrees was very far in distance from the other shapes in the set, suggesting that this shape was especially problematic. The homogeneity score was again high (0.81; $p < 0.0001$), suggesting that participants perceived the shape space very similarly, even though it was not circular.

a. Prototype Shapes



b. Similarity Matrix



c. MDS Reconstructed Space

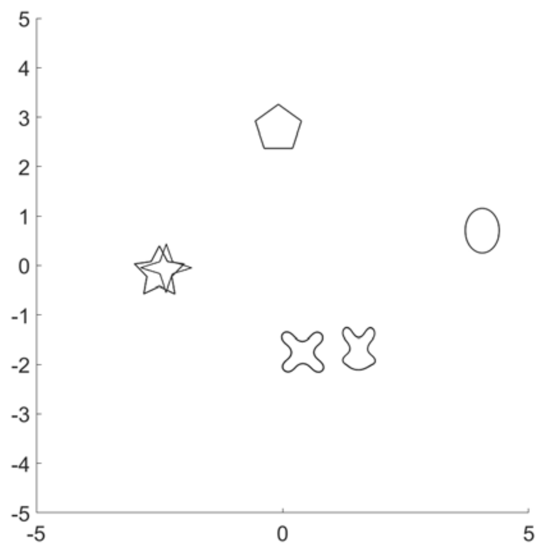


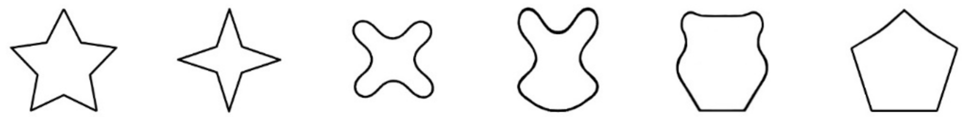
Figure S4. Validation iteration 2 results. (*a*) The set of six rated shapes, which were also prototypes. (*b*) Similarity matrix of averaged similarity ratings from 0 being "*no similarity*" to 5 being "*identical*". (*c*) The MDS reconstructed shape space.

Validation Iteration 3

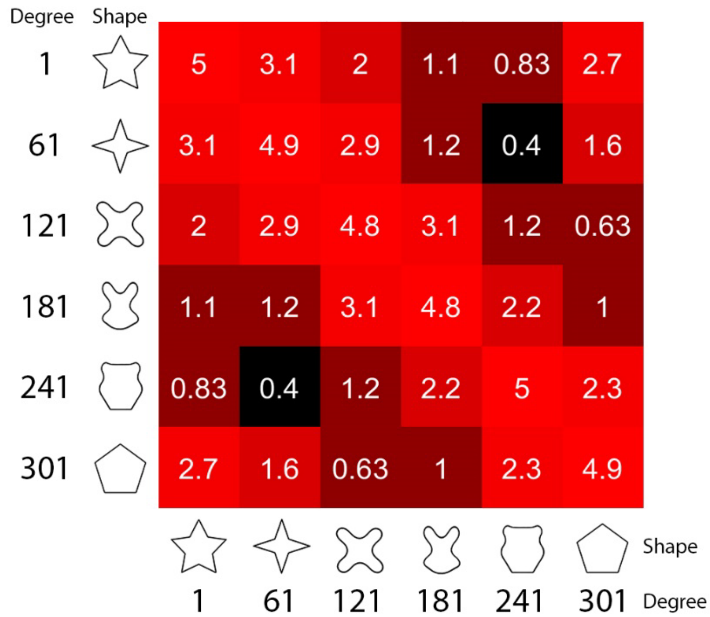
From the similarity matrix and MDS solution in the previous validation iteration, we identified that the oval shape positioned at 241 degrees was especially problematic. This shape was removed and replaced with a prototype that was specifically designed to be closely intermediate in similarity between the shape at 181 degrees and 301 degrees on validation iteration 2 (*Figure S5a*). Additionally, the shape at 301 degrees was manually redesigned to be more similar to the shape at 1 degree. Similarity judgments were collected from six participants (M_{age} = 20.5 years, SD_{age} = 1.76 years, *Female* = 5) on this set of shapes, requiring 72 trials. All combinations of shape pairs were rated relative to each shape in the set, such that each unique pair of shapes were rated 8 times.

Results. C was 0.889 ($p < 0.0001$), suggesting that the shape space in this validation iteration was close to exceeding our threshold of 0.90. Qualitative observation for both the similarity matrix (*Figure S5b*) and resultant MDS solution (*Figure S5c*) revealed that this shape space was approximately circular. The homogeneity score was very high (0.90; $p < 0.0001$), suggesting that participants were homogenous in how they perceived this shape space. Although the shape space in this validation iteration was nearly circular, only six shapes were rated. Thus, we increased the number of rated shapes in the next iteration.

a. Prototype Shapes



b. Similarity Matrix



c. MDS Reconstructed Space

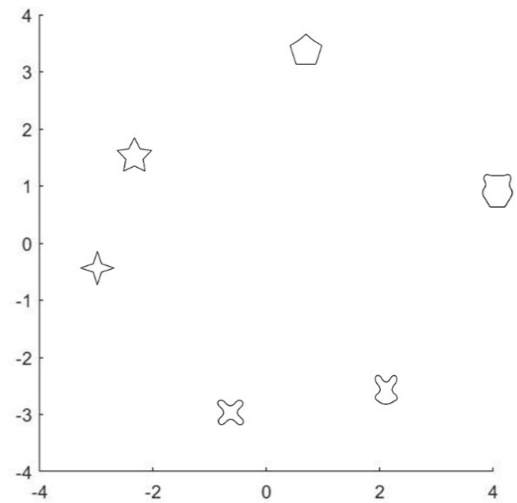


Figure S5. Validation iteration 3 results. (*a*) The set of six rated shapes. (*b*) Similarity matrix of averaged similarity ratings from 0 being “*no similarity*” to 5 being “*identical*”. (*c*) The MDS reconstructed shape space.

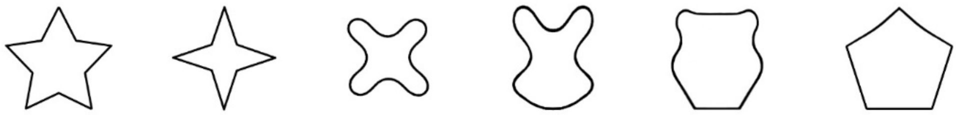
Validation Iteration 4

We identified six shapes in the third validation iteration that were close to being perceptually uniform. In validation iteration 4, we morphed these six shapes (*Figure S6a*) to generate a circular space consisting of 360 stimuli, each mapped onto a single degree of a 2D circle (see *Experiment 1* in the main text for how this was conducted). Twelve equidistant points that were generated as a result of morphing and that were not previously validated were then sampled from the new circular space for validation. This step allowed a greater number of shapes to be rated by participants, allowing us to fix potentially problematic areas to exceed a C value of 0.90.

The twelve equidistant shapes were divided into two groups (*Figure S6b*), with two of the shapes serving as anchor points for later alignment (see *Experiment 1*. *Step 4* for details on the theoretical rationale behind alignment). Six participants were recruited for group 1 (M_{age} = 20.83 years, *SD* = 1.17 years, *Female* = 5). Only a single participant was recruited for group 2 as the reconstructed shape space in group 1 was found not to be circular, so data collection was ended early. In total, as there were seven rated shapes in a single group, each group required 119 trials. All combinations of shape pairs were rated relative to each shape in the set, such that each unique pair of shapes were rated 10 times.

Results. C was 0.874 for group 1 ($p < 0.001$) and 0.886 for group 2 ($p < 0.01$), suggesting that the shape space for validation iteration 4 was less circular than validation iteration 3. Qualitative observation of the similarity matrix (*Figure S7a*) and MDS solution (*Figure S7b*) for group 1 revealed that this space was elliptical. Turning to the similarity matrix, shapes located at 1 degree and 181 degrees were more dissimilar to each other compared to other shapes in the set. Turning to the MDS plot, the dissimilarity of the shapes at 1 degree and 181 degrees was greater compared to the other axis (i.e. the dissimilarity between shapes located at 91 degrees to shapes at 241 through 301 degrees), resulting in an elliptical space. Consistent with all previous validation iterations, the homogeneity score was high (0.84; $p < 0.0001$), suggesting that participants were very similar in how they perceived the shape space. Only one participant was recruited for group 2, as group 1 was already found to be confounded by visual similarity. However, for this one participant, the similarity matrix (*Figure S7c*) and MDS solution (*Figure S7d*) was problematic in a similar region as in group 1.

a. Prototype Shapes



b. Morphed Space

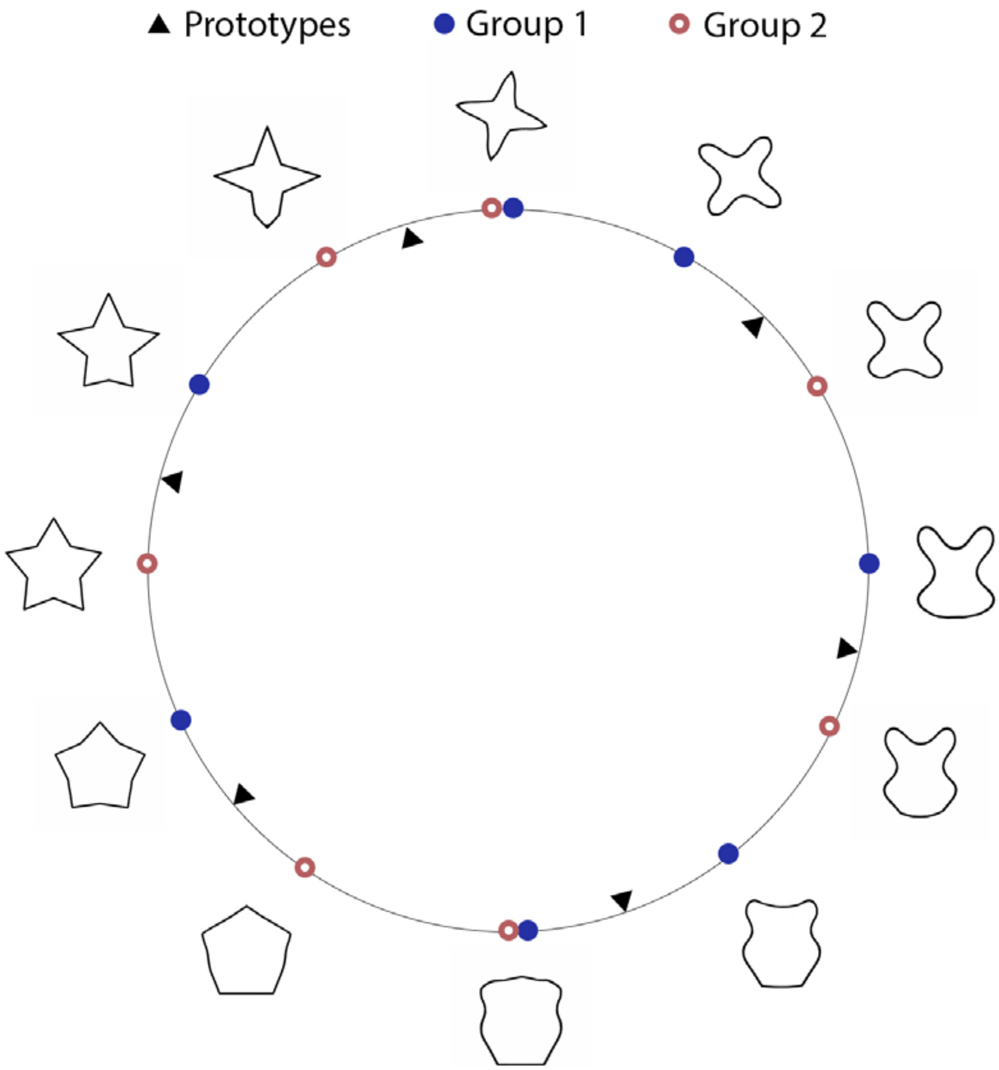


Figure S6. Validation iteration 4. (*a*) The six shapes from validation iteration 3 were used as prototypes. (*b*) Pairs of these prototypes were then morphed together to generate a circular shape space consisting of 360 unique shapes mapped onto each degree of a 2D circle. Shown in this simplified figure are the 12 shapes selected for validation iteration 4, which were different from the prototype shapes used to generate this space (*a*). Shapes denoted with closed blue were rated in group 1, whereas shapes denoted with open red were rated in group 2.

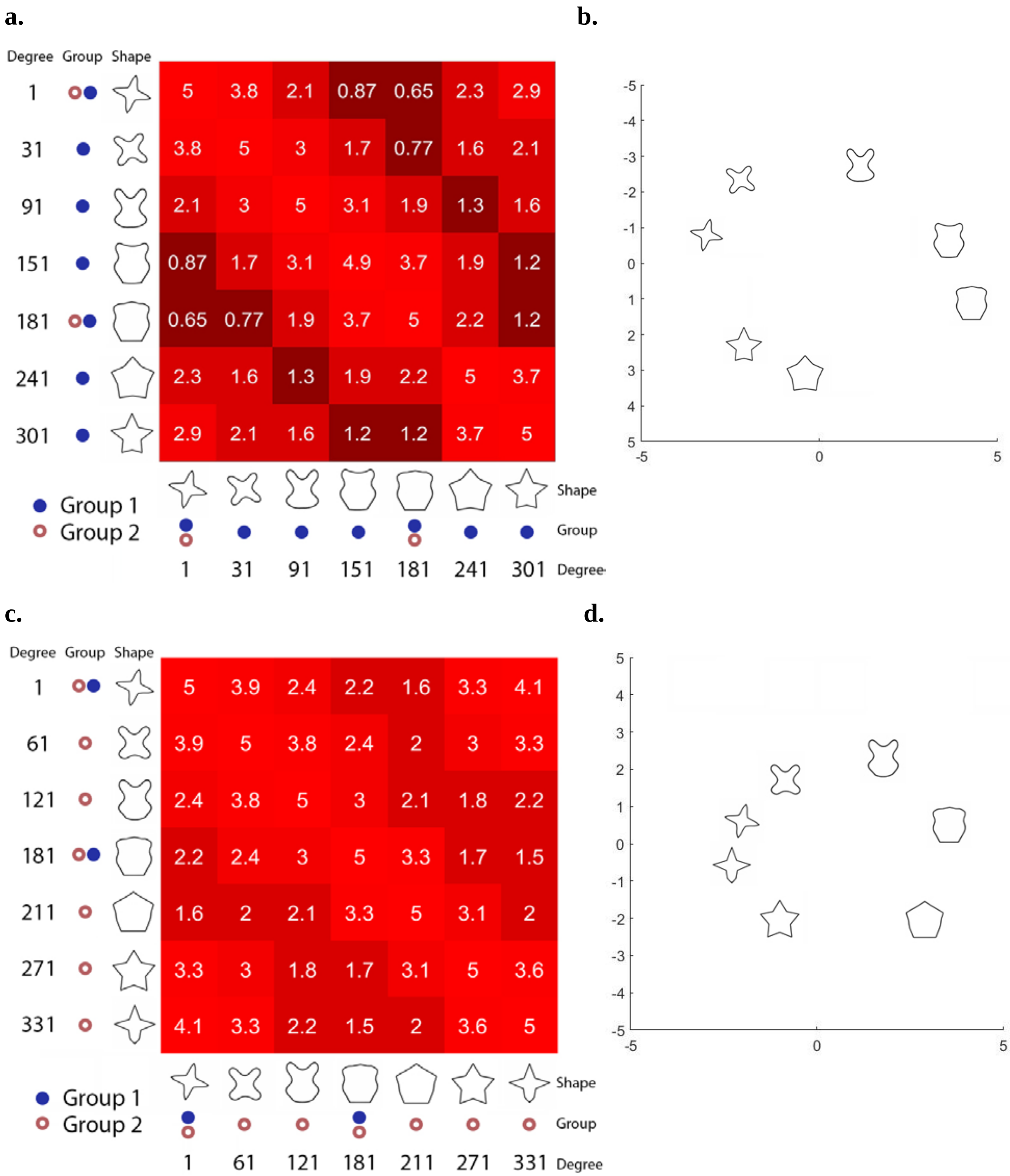


Figure S7. Validation iteration 4 results. The group 1 (*a*) and group 2 (*c*) similarity matrix of averaged similarity ratings from 0 being "*no similarity*" to 5 being "*identical*". The MDS reconstructed shape space for group 1 (*b*) and group 2 (*d*) was elliptical.

Validation Iteration 5

From validation iteration 4, we identified that there was greater dissimilarity between the shapes at 1 degree and 181 degrees compared to the other axis in the shape space (i.e. shapes at 91 degrees and 271 degrees). This imbalance resulted in an elliptical shape space. We redesigned the axis of shape space from 91 degrees to 271 degrees to be more dissimilar by adding entirely new prototype shapes as well as subsequently morphing between prototypes (*Figure S8*). However, as these changes instead drastically reduced the circularity of the reconstructed space (Group 1: $C = 0.783$, $p < 0.01$; Group 2: $C = 0.784$, $p < 0.01$), data collection was ended early. Instead of increasing the dissimilarity between the shapes located at 91 degrees and 271 degrees, the changes in validation iteration 5 unintentionally increased the dissimilarity between the shapes located at 1 degree and 181 degrees, resulting in an even more elliptical space relative to validation iteration 4. For this reason, few participants were recruited for this validation iteration (*Figure S9b*. Group 1, $n = 2$; *Figure S9d*. Group 2, $n = 1$). As this validation iteration had seven rated shapes in the set of each group, each group required 119 trials. All combinations of shape pairs were rated relative to each shape in the set, such that each unique pair of shapes were rated 10 times.

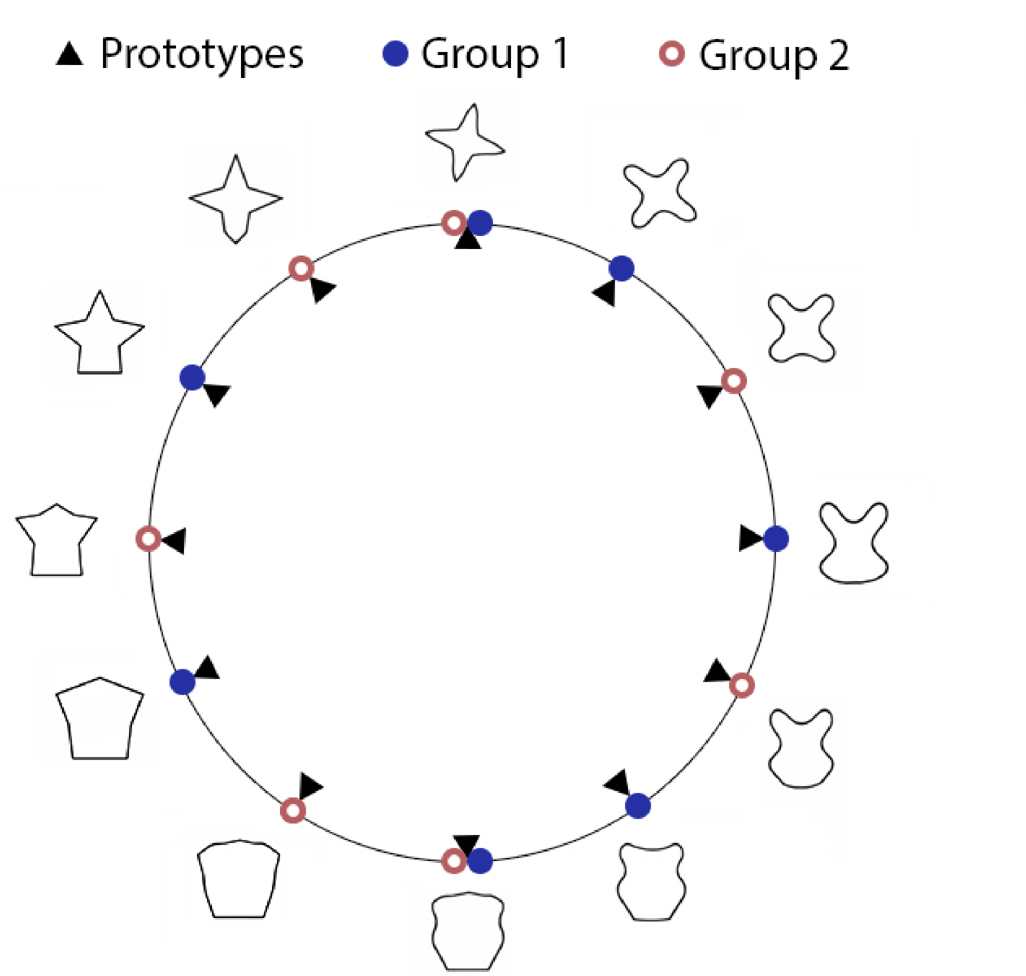


Figure S8. Validation iteration 5. These were the prototype shapes for this iteration, displayed on a 2D circle. Shapes denoted with closed blue were shapes rated in group 1, whereas shapes denoted with open red were shapes rated in group 2.

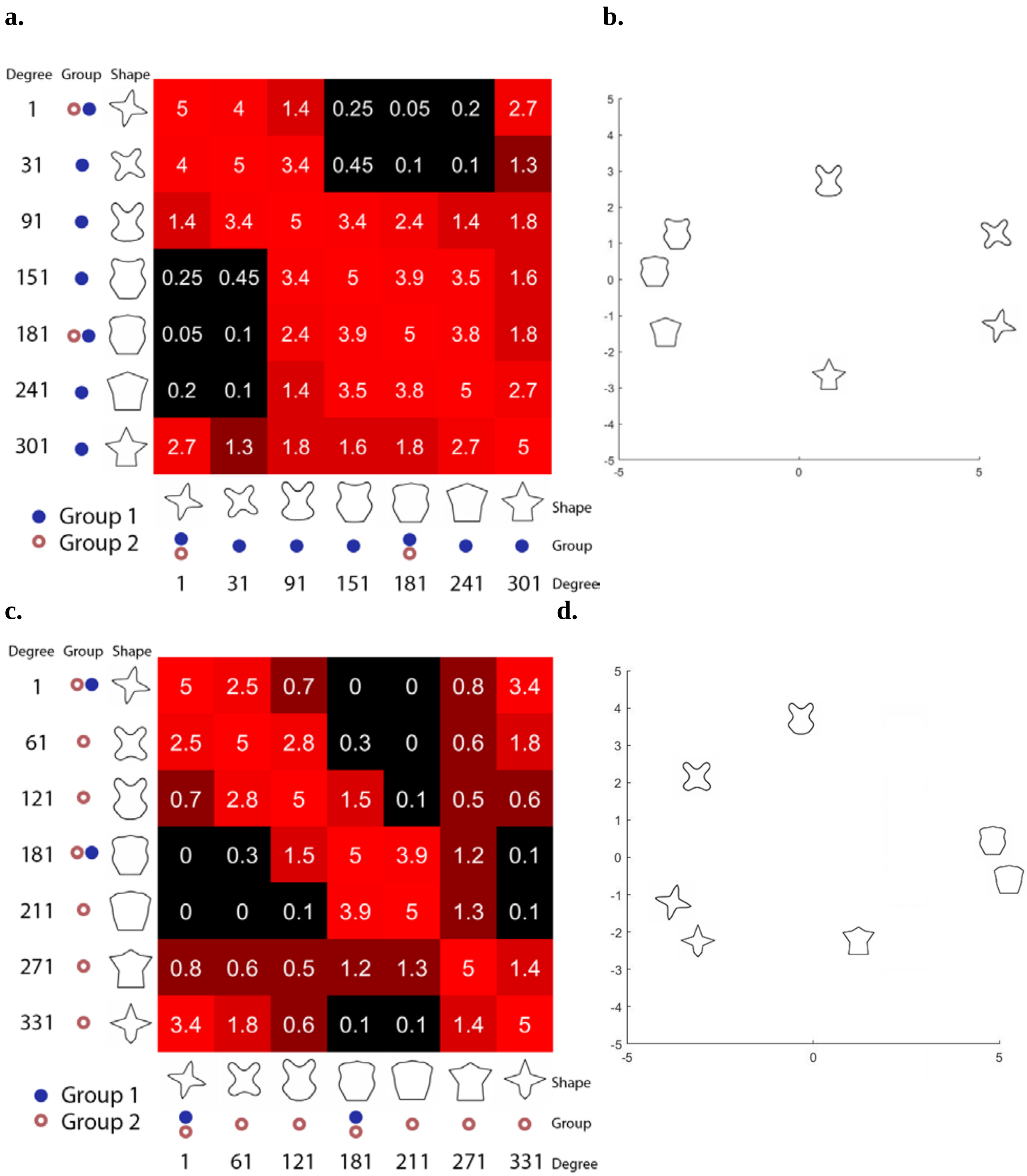


Figure S9. Validation iteration 5 results. The group 1 (*a*) and group 2 (*c*) similarity matrix of averaged similarity ratings from 0 being "*no similarity*" to 5 being "*identical*". The MDS reconstructed shape space revealed an elliptical structure for both group 1 (*b*) and group 2 (*d*).

Validation Iteration 6

From validation iteration 4 and 5, we have identified problematic regions on the shape space that were not circular. More specifically, one axis of the shape space, bounded from the shapes located at 91 degrees and 271 degrees, were more similar relative to the other axis of shape space, bounded from shapes located at 1 degree and 181 degrees. In shape validation 6, we redesigned the shapes located around 271 degrees in order to equate the similarity between the two axis of shape space. This design process included the addition of new prototype shapes as well as morphing.

Participants were again divided into two groups, with each group rating different shapes sampled from alternating locations on shape space (*Figure S10*). Two anchor shapes that were identical in both groups were used to reconstruct the original shape space using alignment. Seven individuals were recruited in group 1 (M_{age} = 21.43 years, *SD* = 1.90 years, *Females* = 6), and eight individuals were recruited in group 2 (M_{age} = 22 years, *SD* = 1.77 years, *Females* = 5). Each group contained seven shapes in the set, with each group requiring 119 trials. All combinations of shape pairs were rated relative to each shape in the set, such that each unique pair of shapes were rated 10 times.

Results. C for group 1 was 0.84 ($p < 0.0001$) and C for group 2 was 0.865 ($p < 0.001$), suggesting that validation iteration 6 was not circular. Qualitative observation of the similarity matrices and MDS reconstructed shape space for group 1 (*Figure S11a, b*) and group 2 (*Figure S11c, d*) revealed that both spaces were elliptical. Turning to the similarity matrices, we again find the axis of shapes located at 91 degrees and 271 degrees was more similar than the axis of shapes located at 1 degree and 181 degrees. This difference in similarity between the two axis of shape space manifested as an ellipse on the MDS solutions.

We then aligned the two MDS reconstructed shape spaces from group 1 and group 2 using Procrustes transformation (*Figure S12*). With two anchor shapes, Procrustes transformation can perfectly align two spaces together. However, we incidentally realized that MDS arbitrarily defines each axis of the shape space during reconstruction, such that the solution of one group may be flipped relative to the solution of a second group. Aligning multiple groups in two dimensions using two anchor points does not take into consideration the arbitrary nature of these axes. In this validation iteration, we flipped one of the axis prior to alignment. In validation iteration 7 however, we solved this problem by using three anchor points rather than two for alignment. For these reasons, we calculated the circularity value of the Procrustes aligned space for this validation iteration. In the final validation iteration, we calculated the circularity value for the affine aligned space, as affine alignment can perfectly align three anchor points in two dimensions (see *Experiment 1. Step 4 – Reconstructing shape space*).

C of the aligned shape space was 0.866 ($p < 0.0001$), suggesting that validation iteration 6 was almost circular. Qualitative observation of the aligned MDS reconstructed shape space revealed that the shape positioned on 301 degrees was especially problematic. This problematic shape was redesigned in the final validation iteration and replaced with a new prototype. Consistent with all the previous validation iterations, participant shape spaces were found to be highly correlated in both group 1 (homogeneity score = 0.78; $p < 0.0001$) and group 2 (homogeneity score = 0.83; $p < 0.0001$).

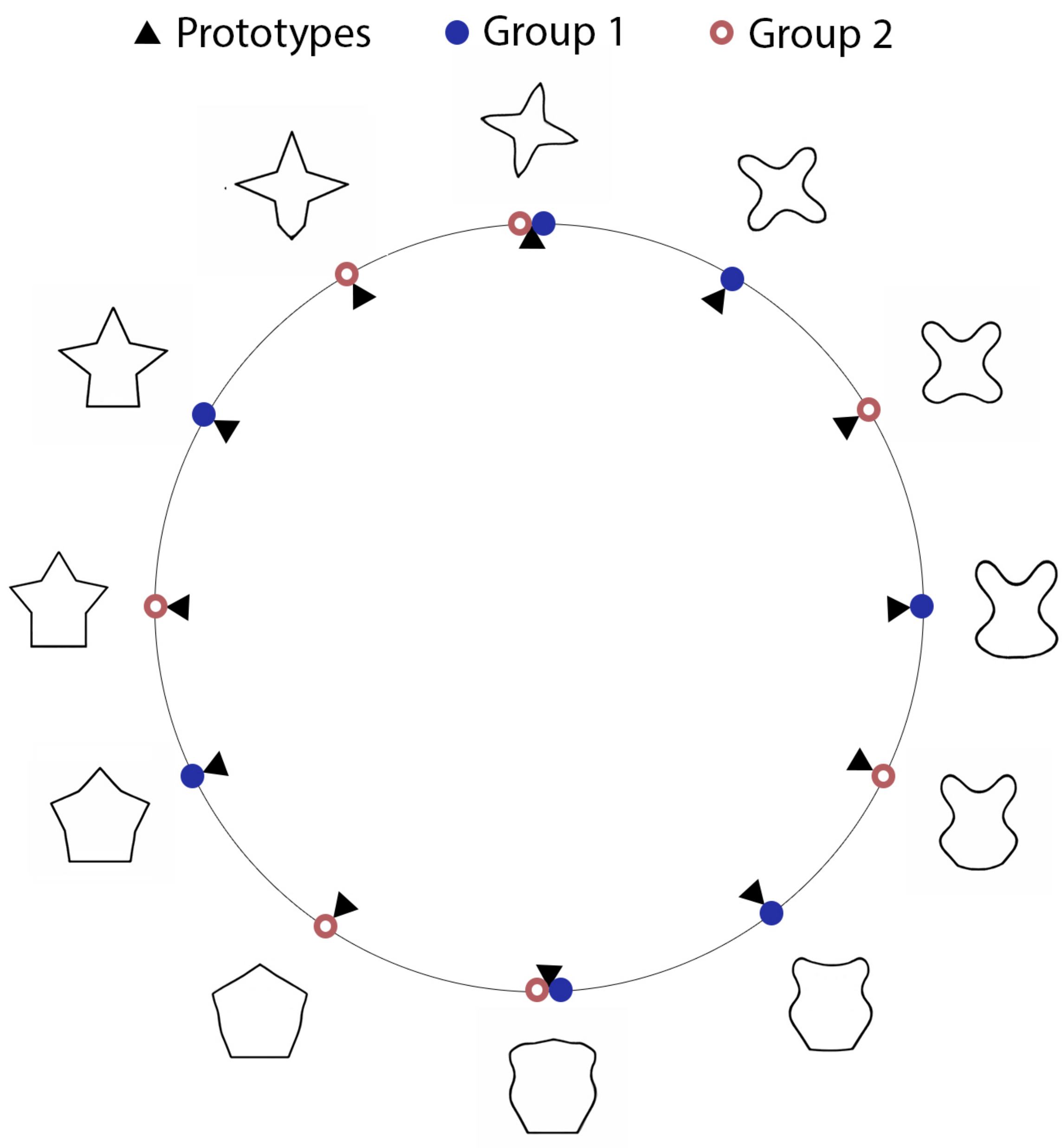


Figure S10. Validation iteration 6. These were the prototype shapes for this iteration, displayed on a 2D circle. Shapes denoted by closed blue were rated in group 1, whereas shapes denoted by open red were rated in group 2.

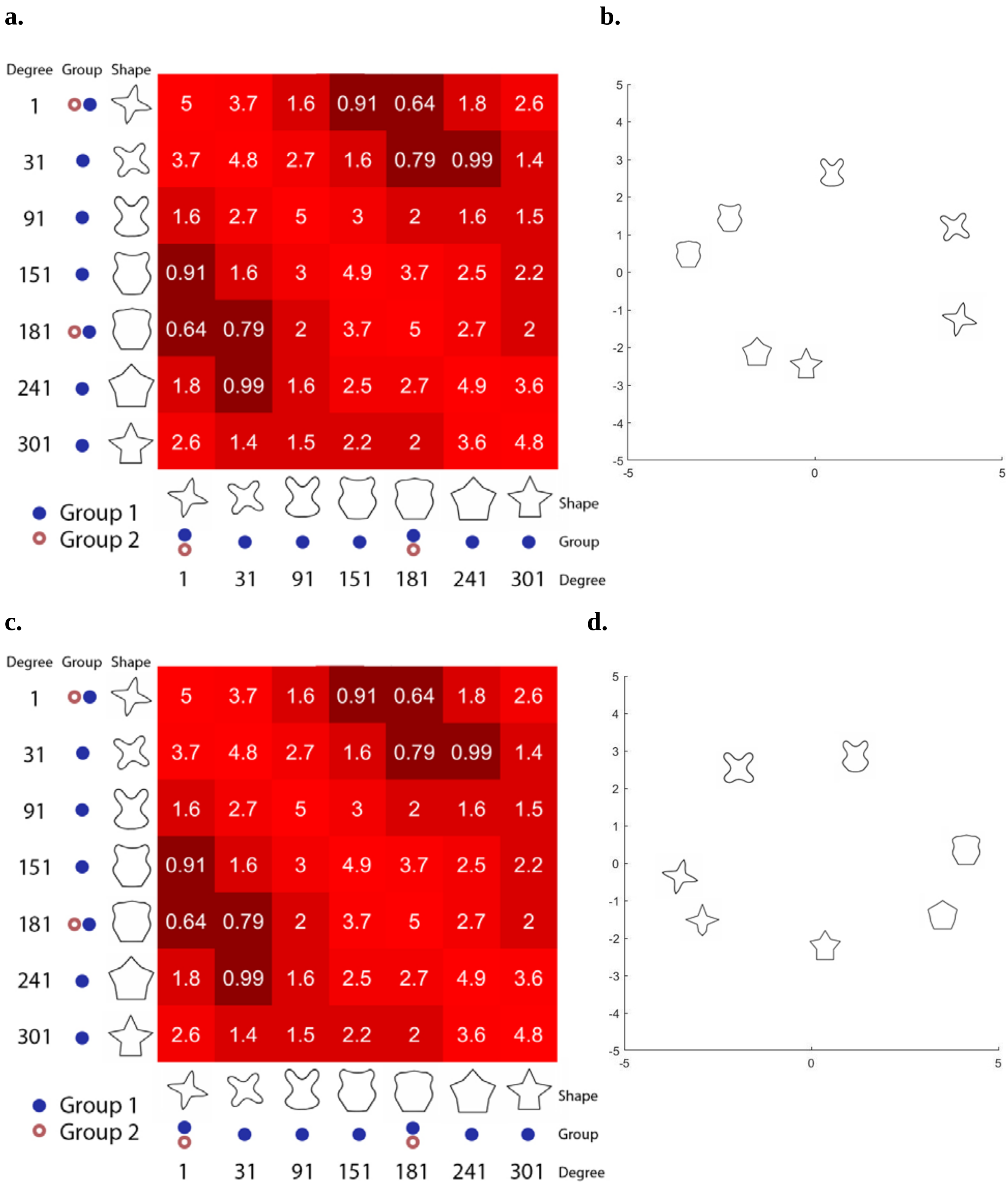


Figure S11. Validation iteration 6. The group 1 (*a*) and group 2 (*c*) similarity matrix of averaged similarity ratings from 0 being "*no similarity*" to 5 being "*identical*". The MDS reconstructed shape space revealed an elliptical structure for both group 1 (*b*) and group 2 (*d*).

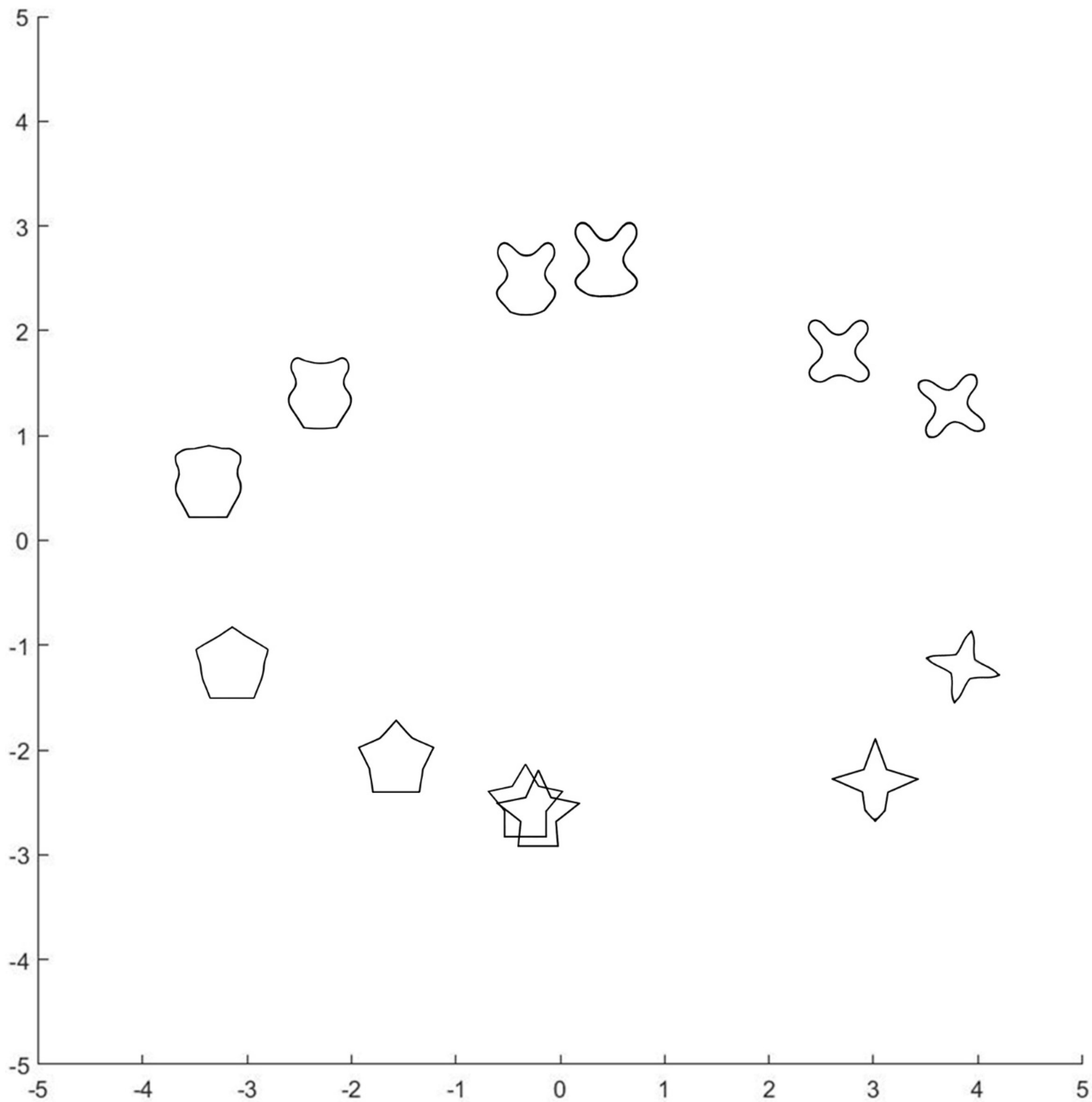


Figure S12. Alignment results for validation iteration 6. The aligned shape space was ellipitical, with one region on this space especially confounded by similarity.

Validation Iteration 7

The problematic region identified in validation iteration 6 was fixed by redesigning one prototype shape located at 301 degrees, whereas all other prototypes remained identical (*Figure 2*). See the main text for the methodological description of this final validation iteration. Here we report the group 1 and group 2 results.

Results. Group 1 and group 2 exceeded our threshold of C = 0.90, suggesting that the reconstructed shape space for both groups were circular. C for group 1 was 0.92 (p < 0.0001 using permutation testing), whereas C for group 2 was 0.93 (p < 0.0001 using permutation testing). Qualitative observation of both the group similarity matrices and MDS reconstructed shape spaces supported the quantitative circularity scores (*Figure S13*). Turning to the group similarity matrices, shapes located closer in angular distance on the original shape space were rated more similarly compared to shapes located further in angular distance (*Figure S13a, c*). Turning to the MDS reconstructed shape spaces, the positions of the vertices approximately matched the positions of items sampled from the original shape space (*Figure S13b, d*).

To visualize the two group matrices together, we combined them by populating a single matrix with the elements from both groups (see main text; *Figure 8*). The ratings of anchor shapes relative to other anchor shapes were the only shapes with ratings in both groups, and so these ratings were averaged to produce a single value.

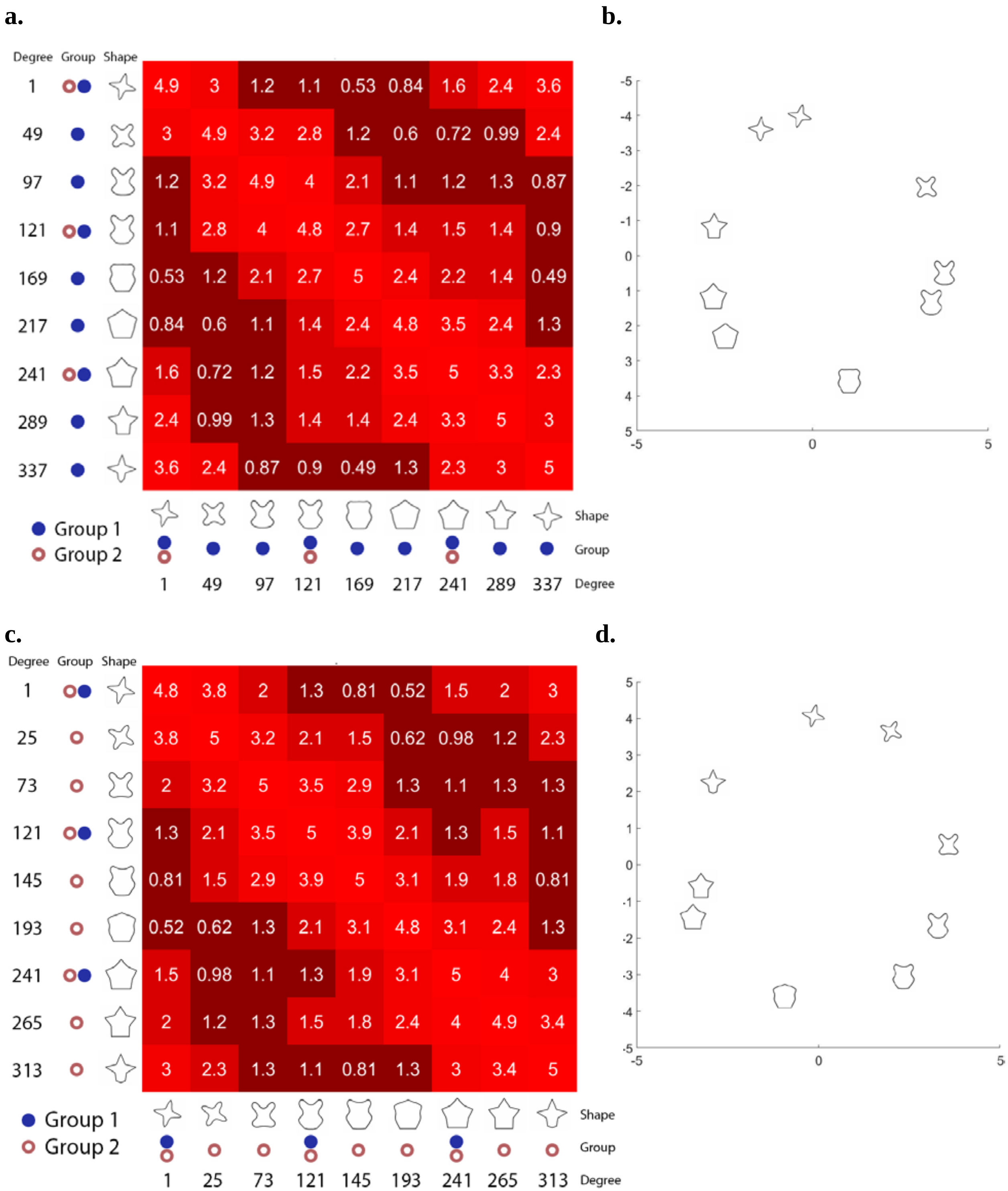


Figure S13. The group 1 (*a*) and group 2 (*c*) similarity matrix of averaged similarity ratings from 0 being "*no similarity*" to 5 being "*identical*". Pairs of shapes closer in angular distance were rated more similarly than shapes further away in angular distance. The MDS-reconstructed shape space for group 1 (*b*) and group 2 *(d)* was circular and approximated VCS space (shown in *Figure 3b*).